Optimal Algorithm for Online Multiple Knapsack



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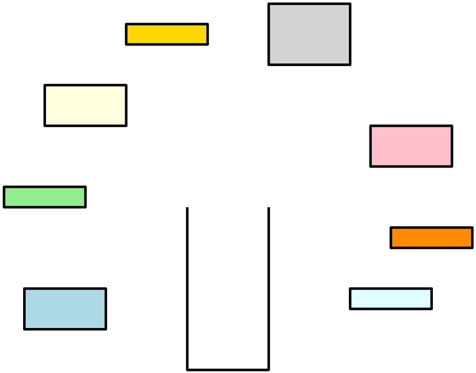
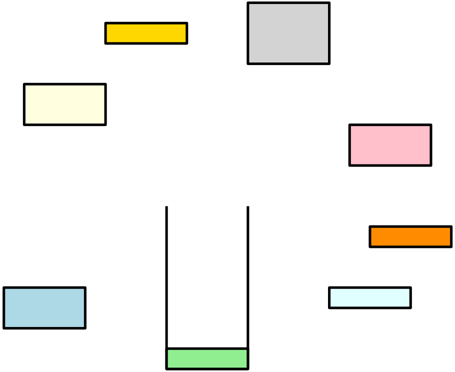
Knapsack



Knapsack



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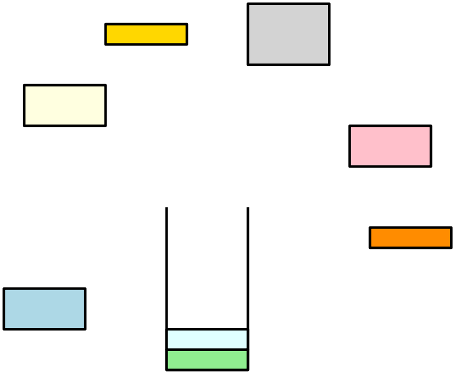
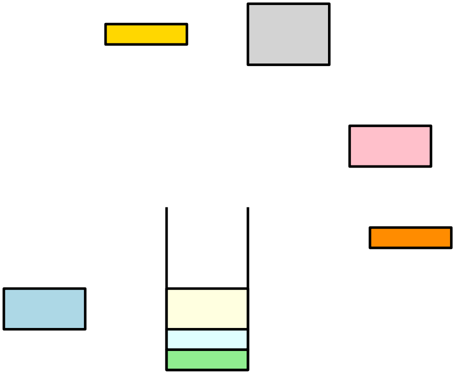
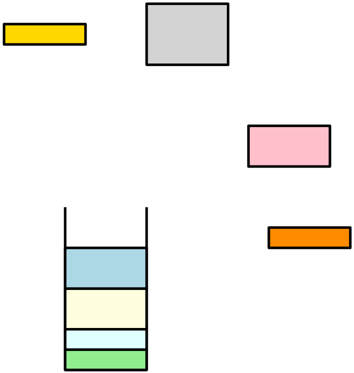


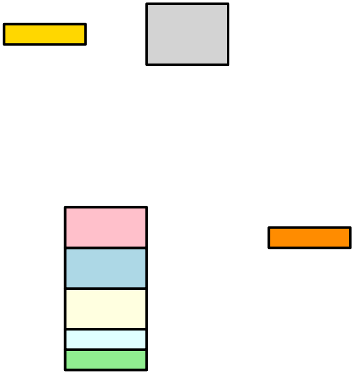
Knapsack



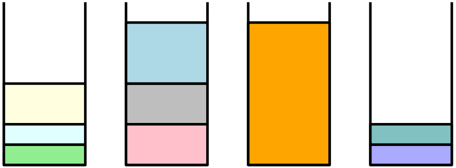
Knapsack



Knapsack



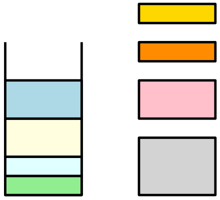
Multiple Knapsack



Textbook Knapsack (offline)

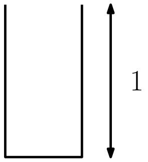
Given

- one knapsack of capacity 1
- multiset of items (size and weight)

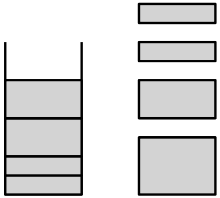


Choose a subset of items

- sum of sizes ≤ 1
- maximize total weight



Proportional Knapsack (offline)

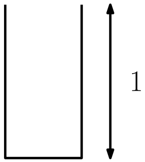


Given

- one knapsack of capacity 1
- multiset of items (size ~~and weight~~)

Choose a subset of items

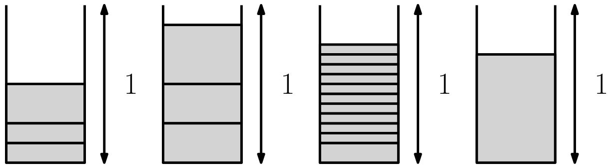
- sum of sizes ≤ 1
- maximize total ~~weight~~ size



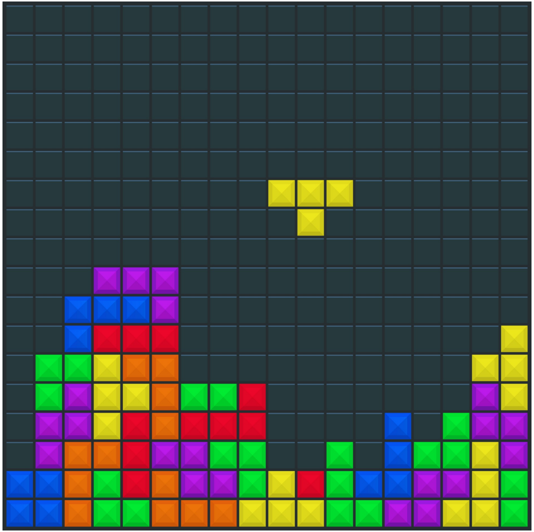
Multiple Knapsack (offline)

Choose a subset of items

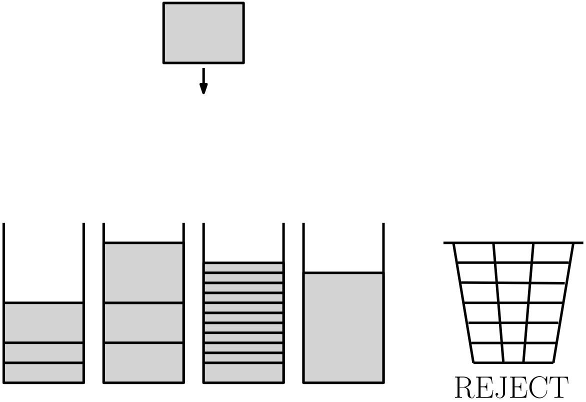
- assign accepted items to a knapsacks
- in each knapsack: total size of items ≤ 1
- maximize total size



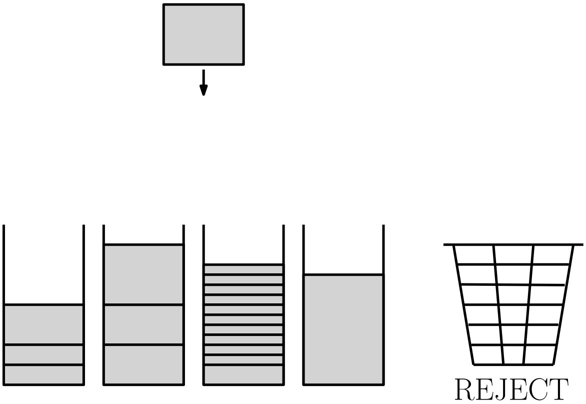
Online



Online Multiple Knapsack



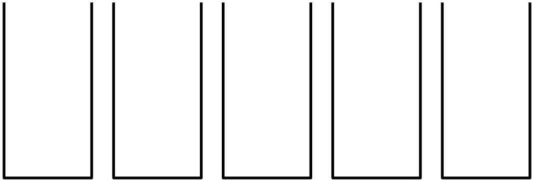
Online Multiple Knapsack



maximize $\frac{ALG_{online}}{OPT_{offline}}$

Known results

FirstFit is 0.5-competitive (Cygan et al. [TOCS 2016])



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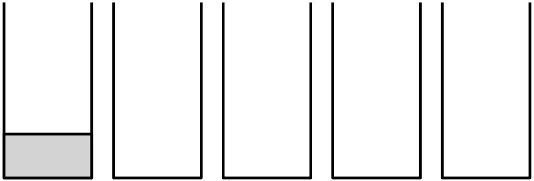

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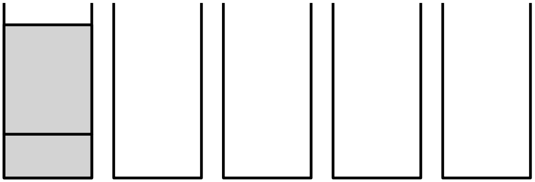

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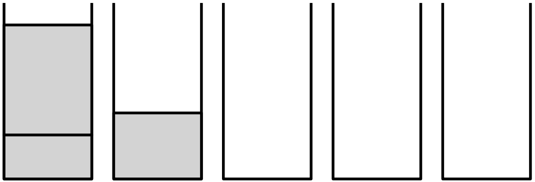

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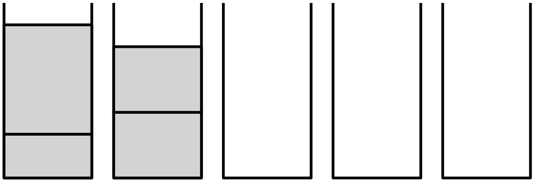

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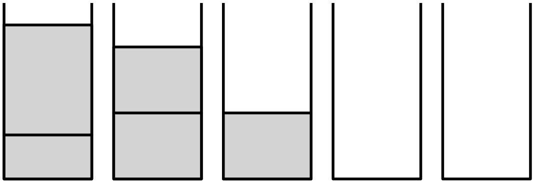

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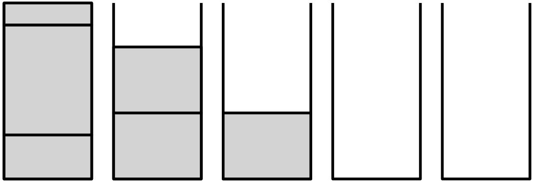

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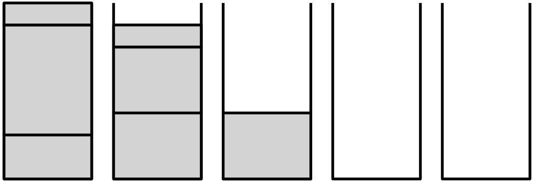

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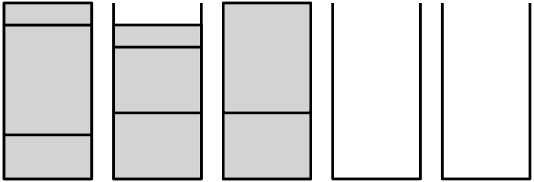

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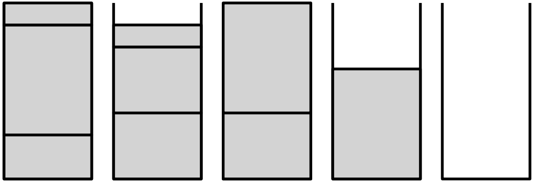

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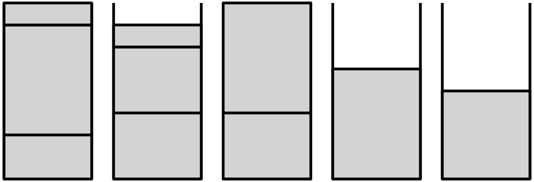
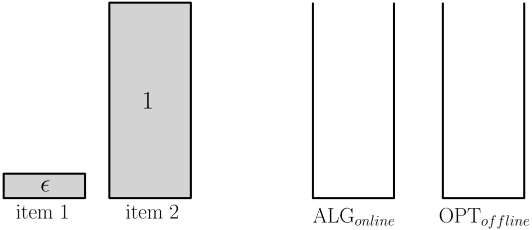


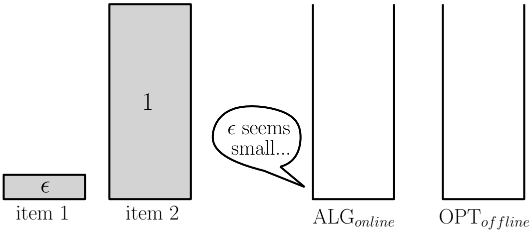
Known results

Bad news for One Online Knaspack



Known results

Bad news for One Online Knaspack



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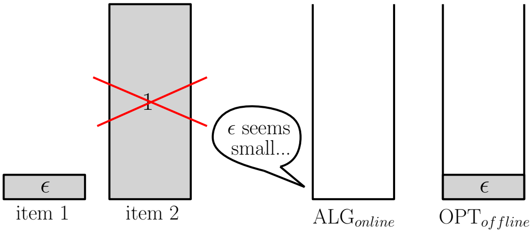

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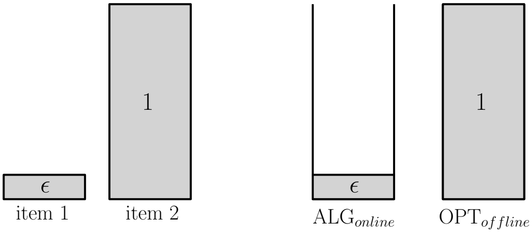


Known results

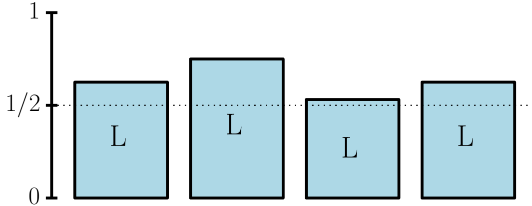
(max objective: higher is better)

Our contributions

(max objective: higher is better)

Rising Threshold Algorithm

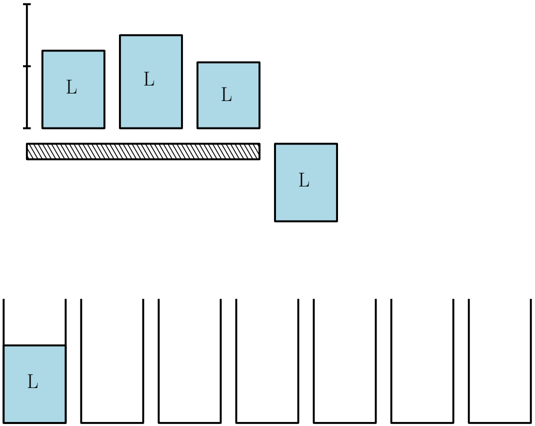
We say that items $(1/2, 1]$ are large



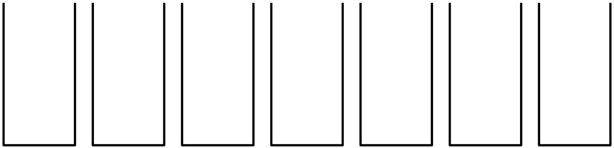
(max 1 large per knapsack)

Rising Threshold Algorithm

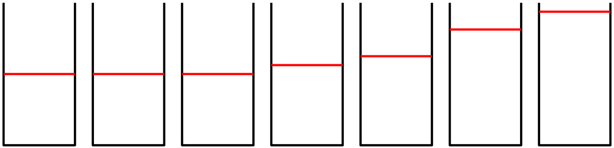
Step 1. Algorithm for large items



Rising Threshold Algorithm (for large items)

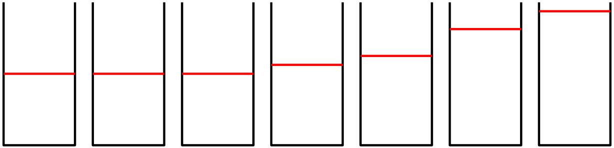


Rising Threshold Algorithm
(for large items)



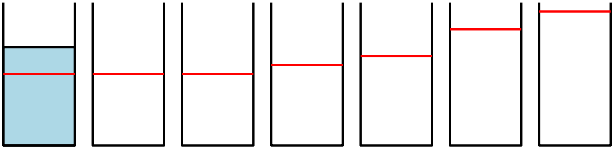
assign each knapsack a threshold (tbd)

Rising Threshold Algorithm (for large items)



- fill from the left
- reject if under threshold

Rising Threshold Algorithm (for large items)



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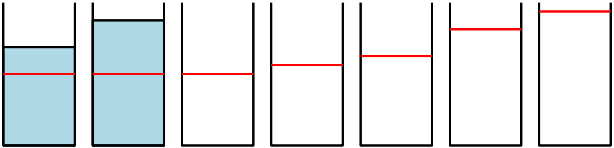

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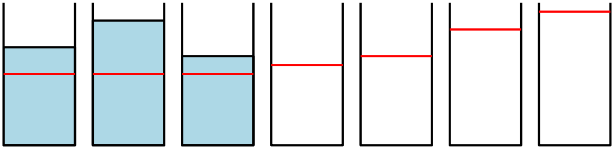

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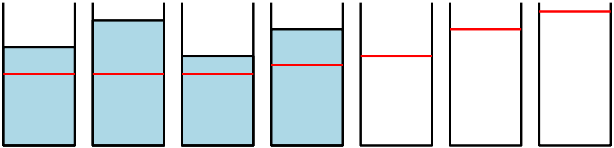

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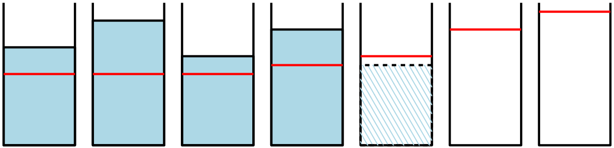

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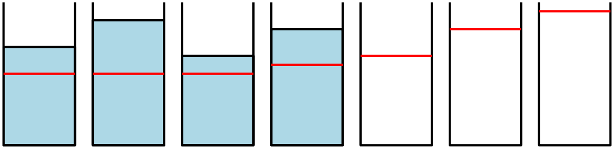

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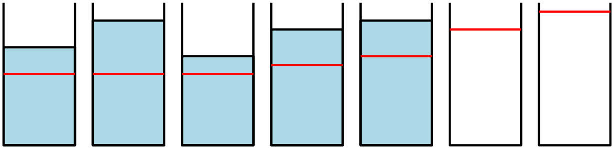

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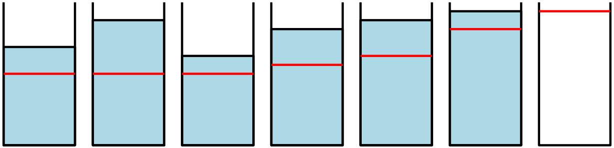

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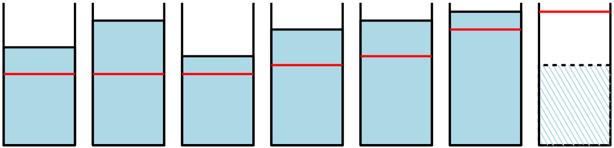

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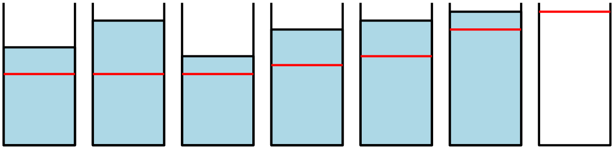

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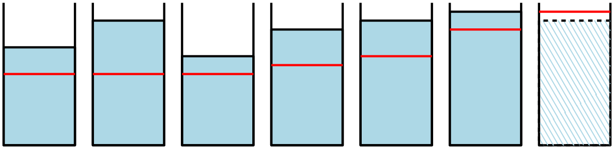

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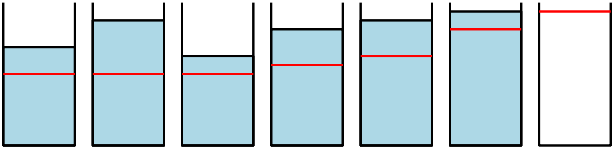
Rising Threshold Algorithm
(for large items)



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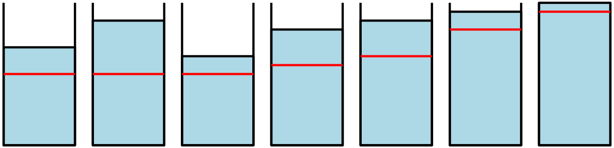

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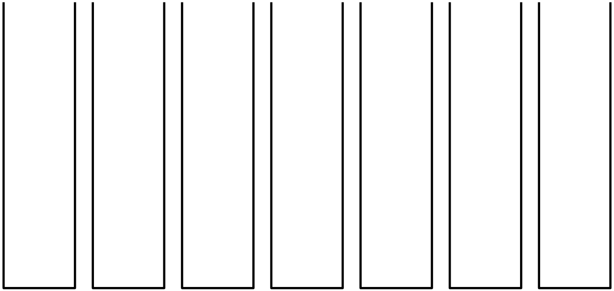
Rising Threshold Algorithm
Threshold function

$$f(x) = \max\{1/2, (2e)^{x-1}\}$$



n knapsacks

Rising Threshold Algorithm
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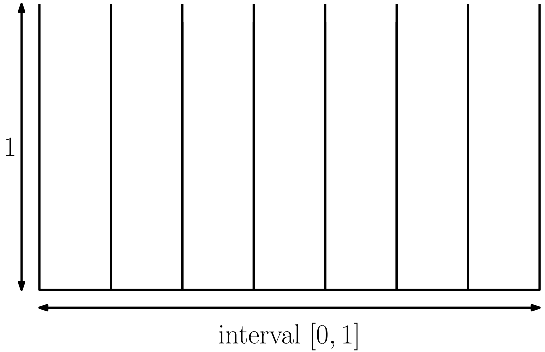

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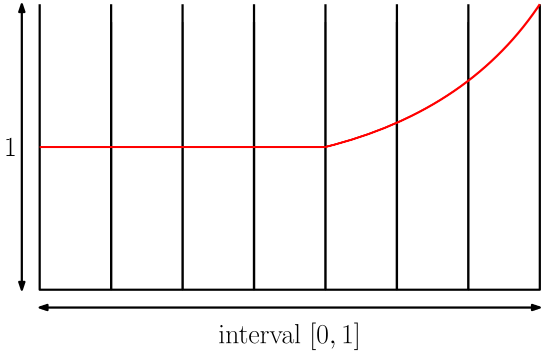

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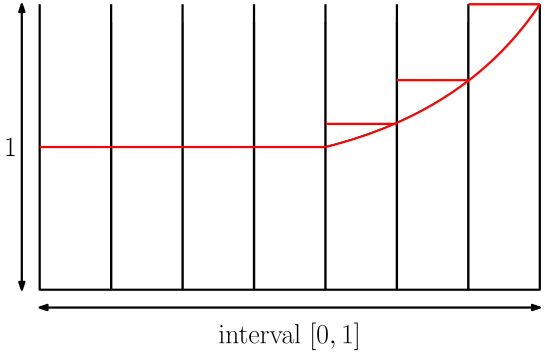
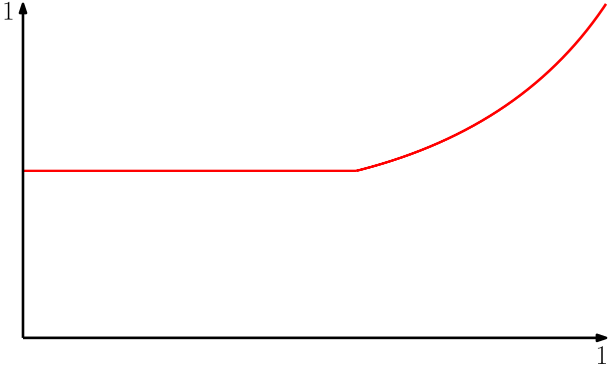


Rising Threshold Algorithm
Basic properties of f

$$f(x) = \max\{1/2, (2e)^{x-1}\}$$



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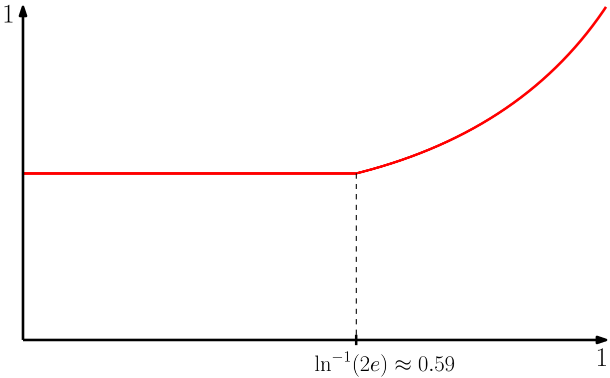

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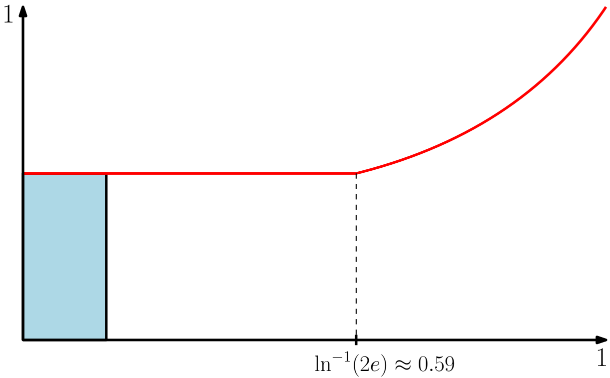

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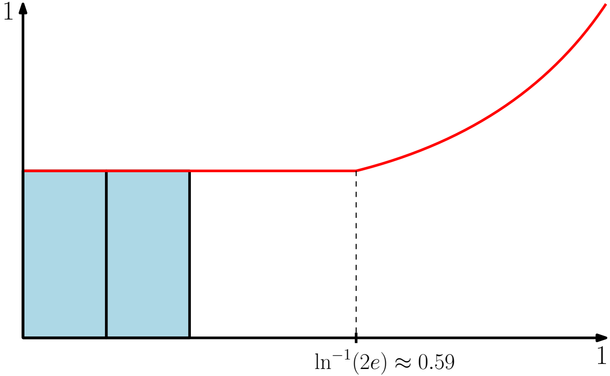

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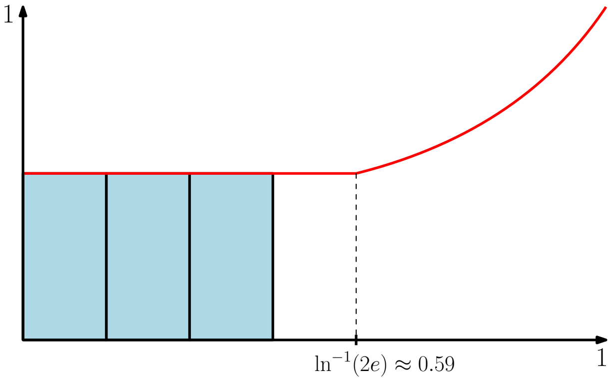

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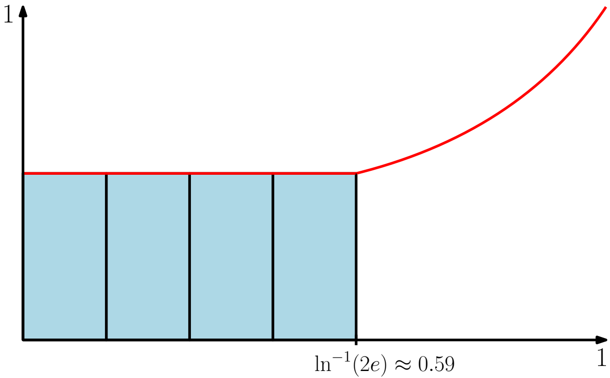

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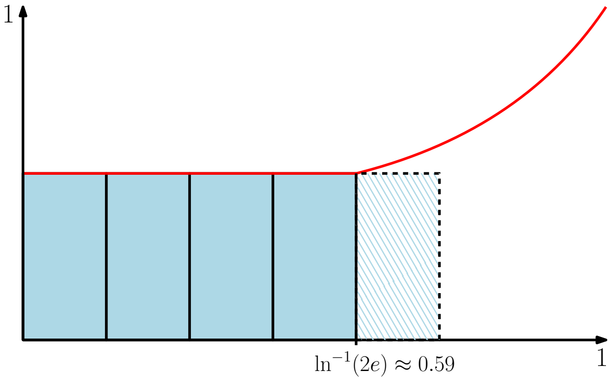

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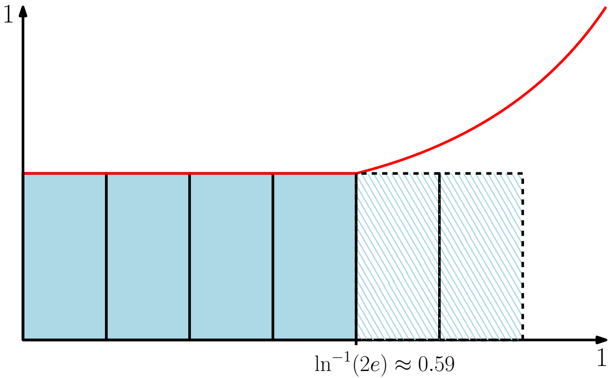

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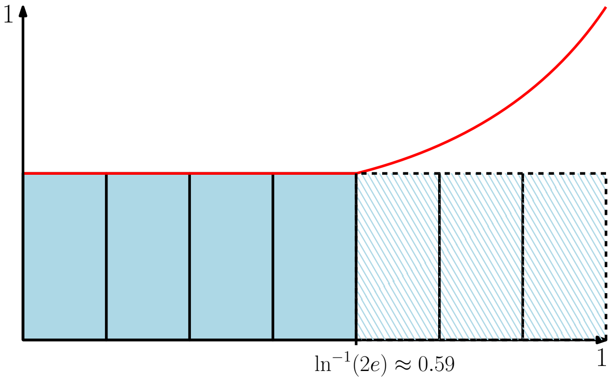
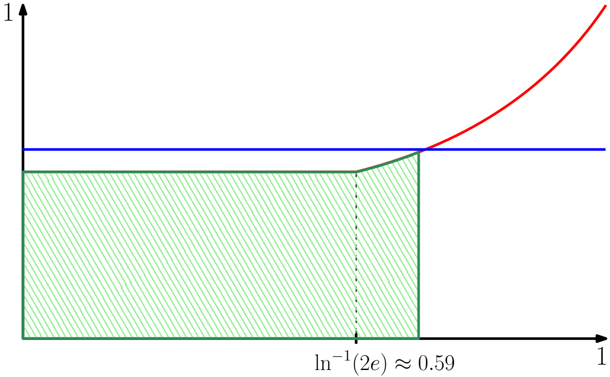


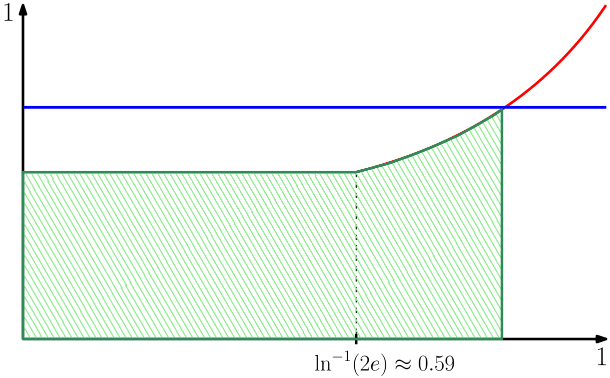
Rising Threshold Algorithm
Most important property of f

$$\frac{\int^{x} f(t)dt}{f(x)\cdot 1} = \frac{\text{green area}}{\text{area below blue}} = \ln^{-1}(2e) \approx 0.59$$



Rising Threshold Algorithm
Most important property of f

$$\frac{\int^{x} f(t)dt}{f(x)\cdot 1} = \frac{\text{green area}}{\text{area below blue}} = \ln^{-1}(2e) \approx 0.59$$



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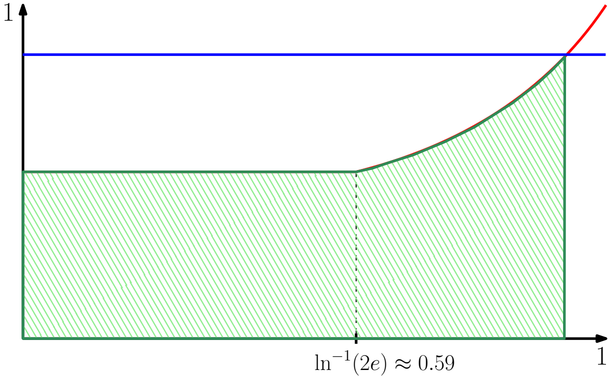

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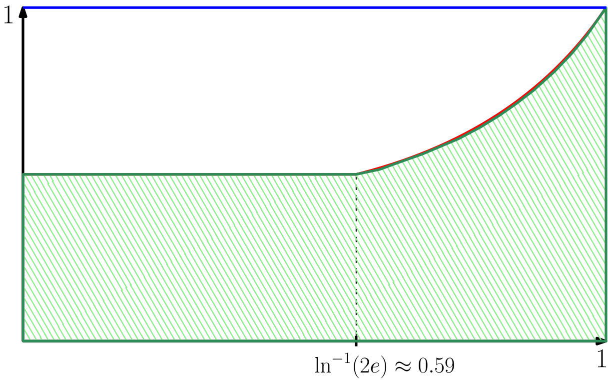


Rising Threshold Algorithm

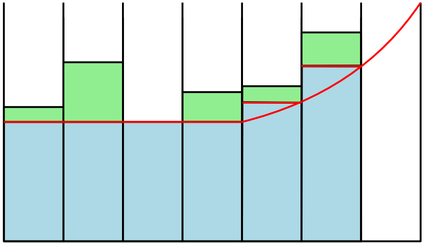
Analysis for large

$$=$$

$$\frac{\int^{x} f(t)dt}{f(x) \cdot 1} \approx 0.59$$

$$+$$

items exceeding threshold benefit both ALG and OPT



Rising Threshold Algorithm
Next steps

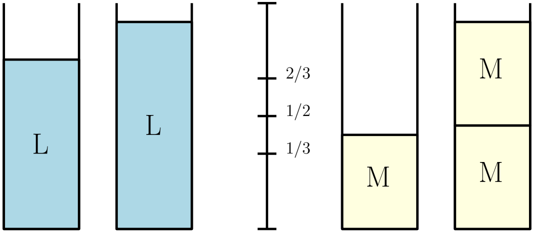
- Step 1. Algorithm for large items $(1/2, 1]$

Rising Threshold Algorithm
Next steps

- Step 1. Algorithm for large items $(1/2, 1]$ $\checkmark$

Rising Threshold Algorithm
Next steps

- Step 1. Algorithm for large items $(1/2, 1]$ ✓
- Step 2. Algorithm for large and medium items $(1/3, 1/2]$:



(two medium items fit in 1 knapsack)

Adding medium items $(1/3, 1/2]$

Algorithm properties

- take large items according to threshold
- never reject medium items

Adding medium items $(1/3, 1/2]$

Algorithm properties

- take large items according to threshold
- never reject medium items

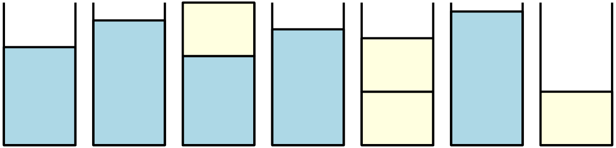
$$\Downarrow$$

Observation. If finished with some empty knapsacks $\Rightarrow$ optimal!

$$\frac{ALG_L + M}{OPT_L + M} \geq \frac{ALG_L}{OPT_L}$$

Adding medium items $(1/3, 1/2]$

- Case 1. Finished with some empty knapsacks ✓
- Case 2. Finished with no empty knapsacks:



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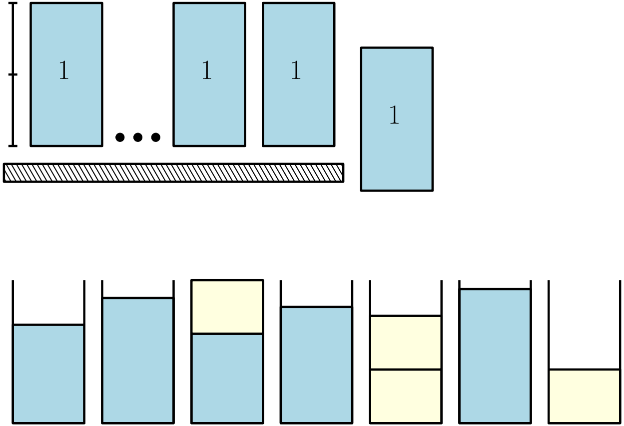

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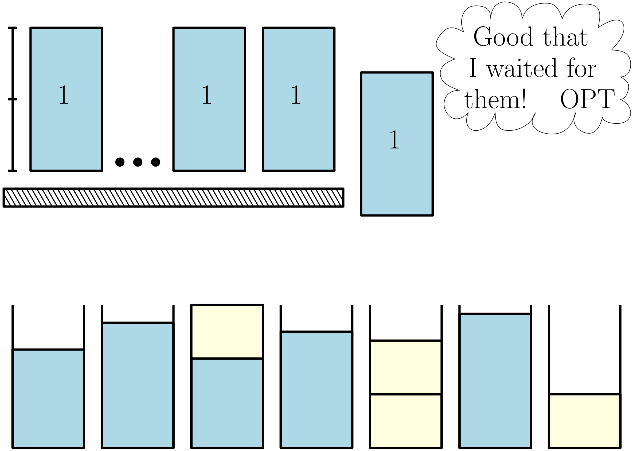
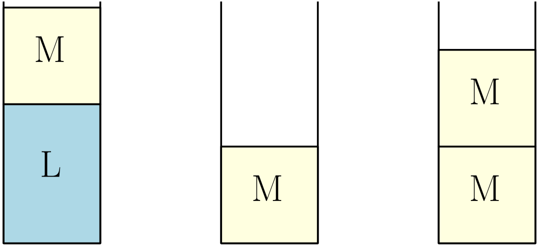
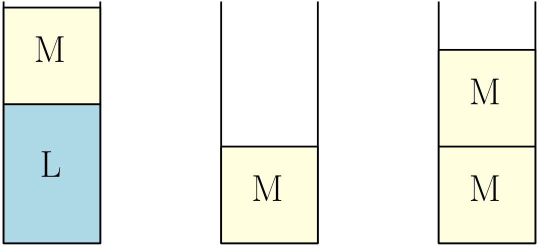


Three options to arrange mediums



combine (with large) | wait (for large) | stack (with medium)

How we arrange mediums



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ALG:

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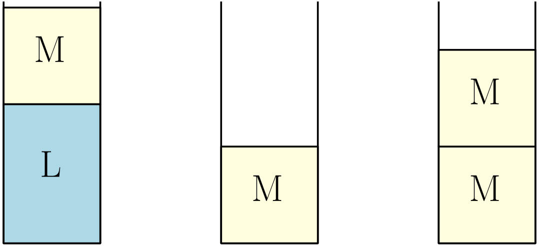

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ALG: $\boxed{\text{always attempt}}$

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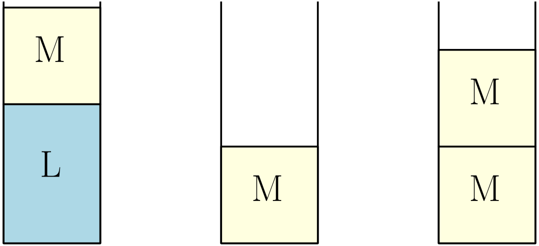
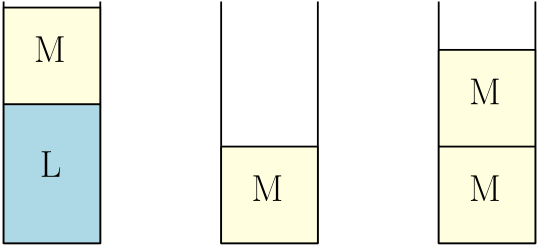


combine	wait	stack
(with large)	(for large)	(with medium)

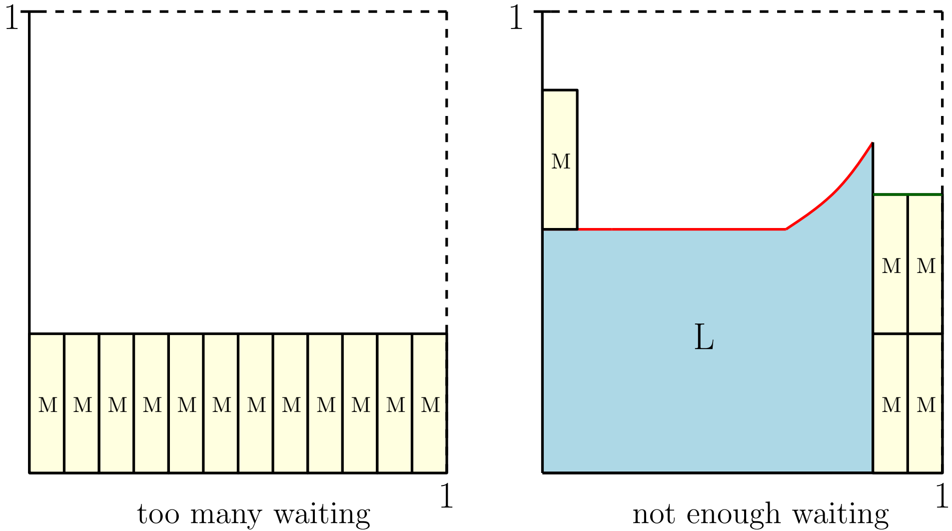
ALG: always attempt | keep some

How we arrange mediums



	combine	wait	stack
	(with large)	(for large)	(with medium)
ALG:	always attempt	keep some	if too many waits

How many medium items should wait?



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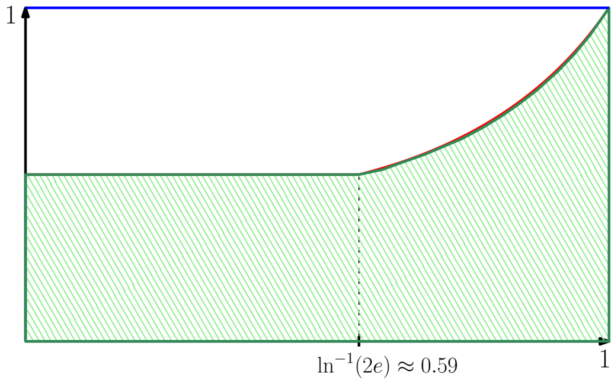

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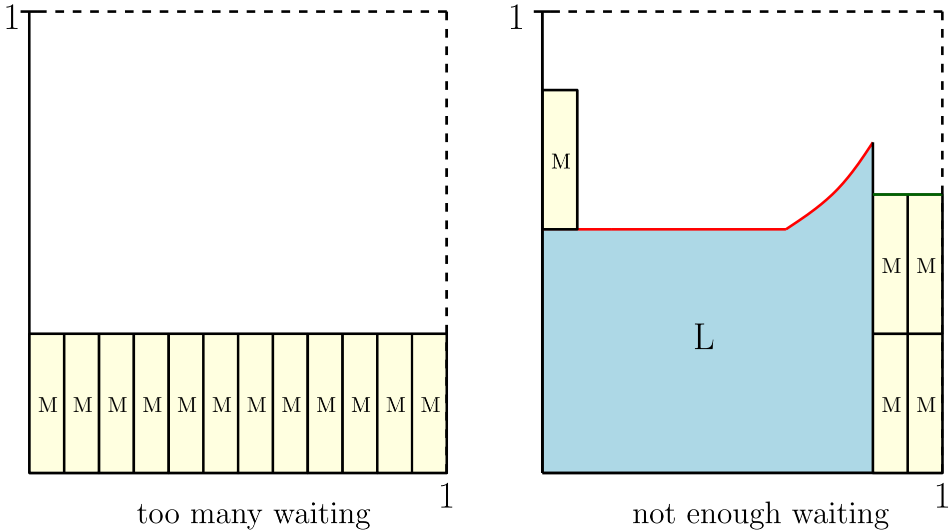

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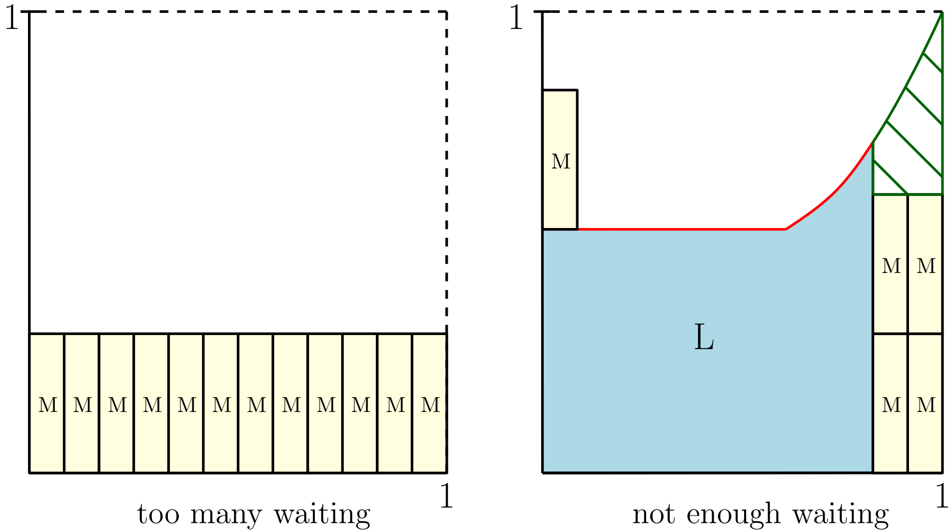

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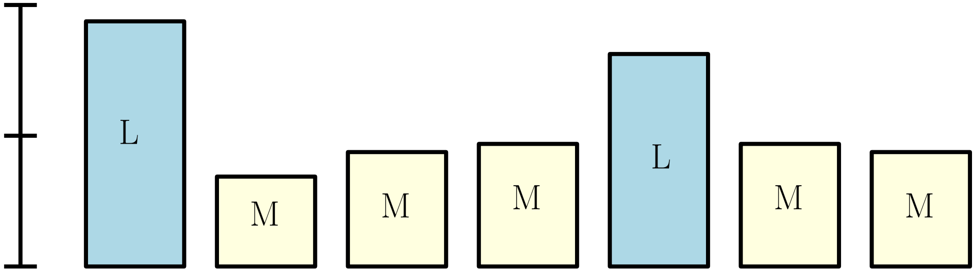

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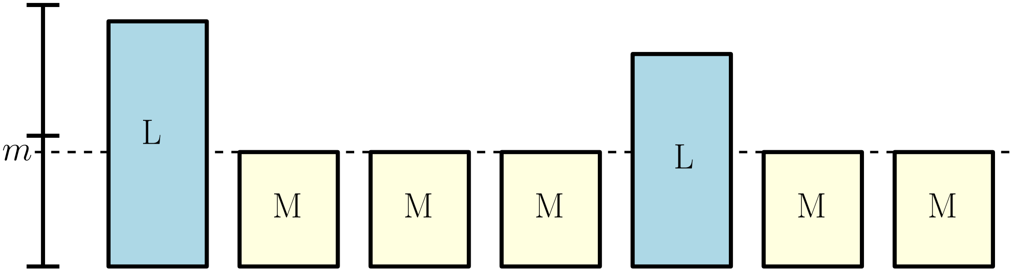


Simplify: all mediums of size $m \in (1/3, 1/2]$

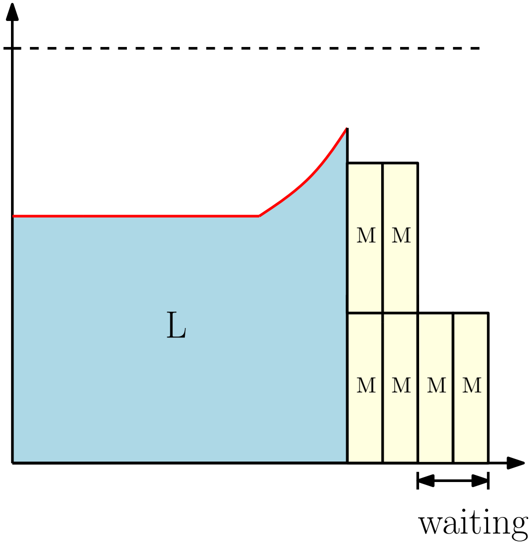
How many medium items should wait?

Answer: to have gain ≥ 0.59

How many medium items should wait?

Each medium item is size m

- gain on waiting $= m$
- gain on stacked $= 2m$
- gain on large $\geq 1 - m$



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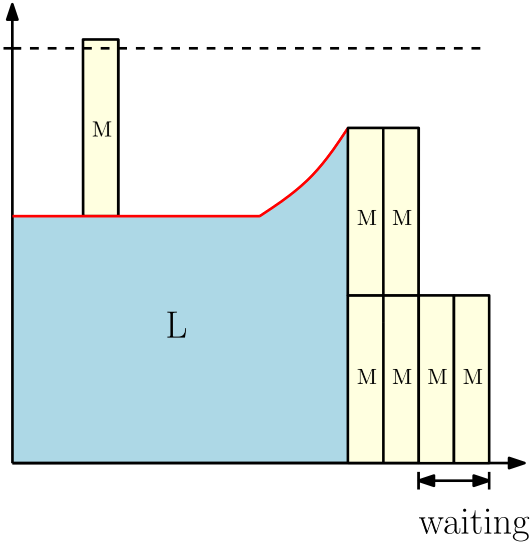

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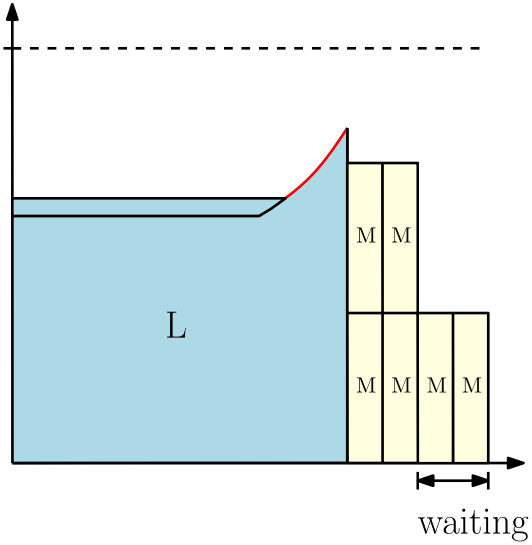

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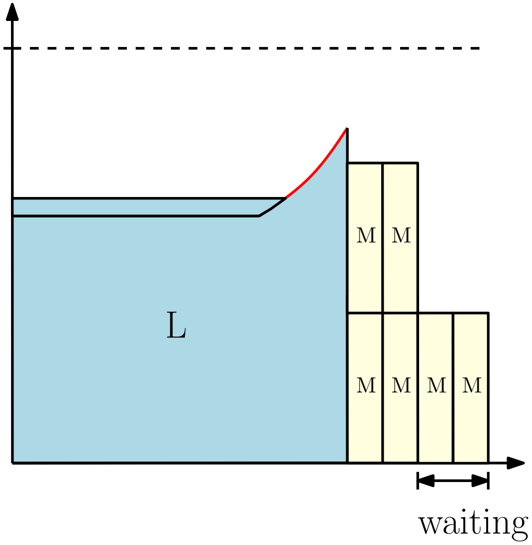

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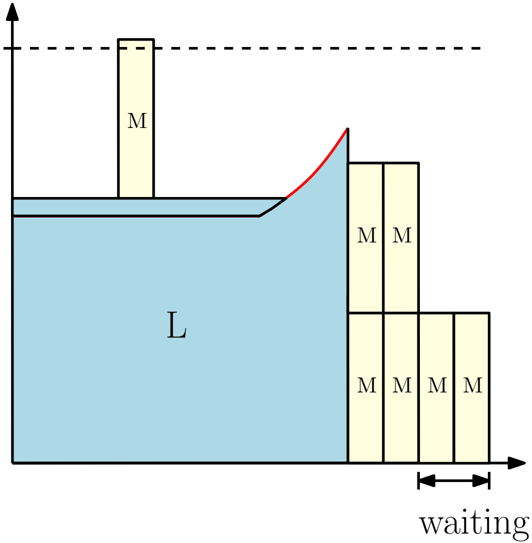

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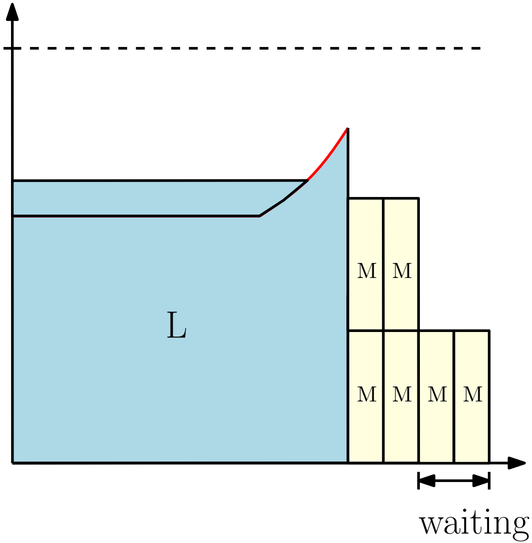

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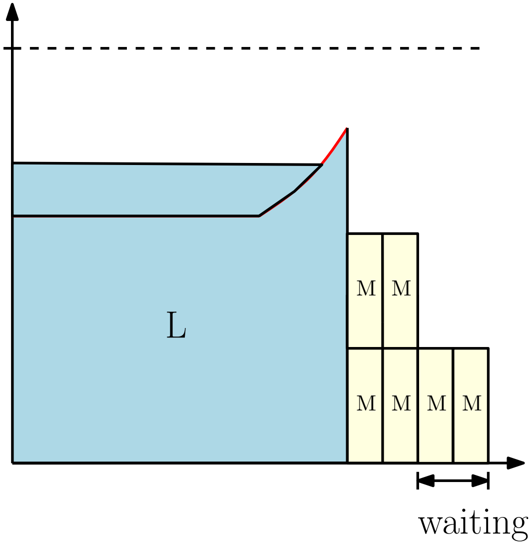
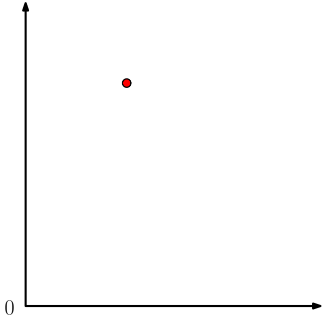


How many medium items should wait?

Answer: fix m, solve for gain(#waiting) ≥ 0.59



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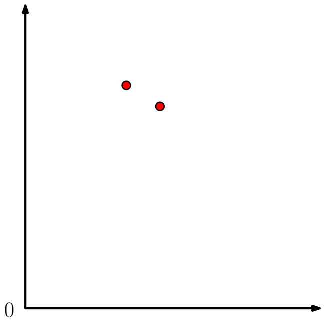

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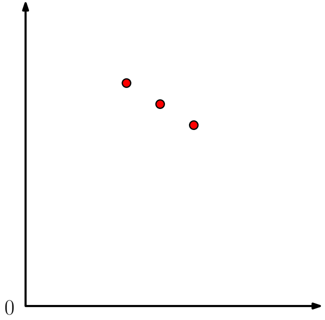

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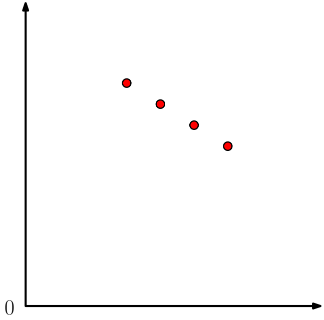

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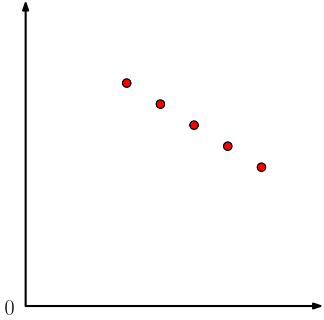

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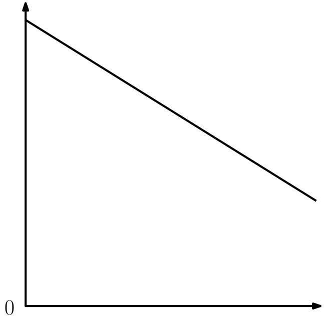
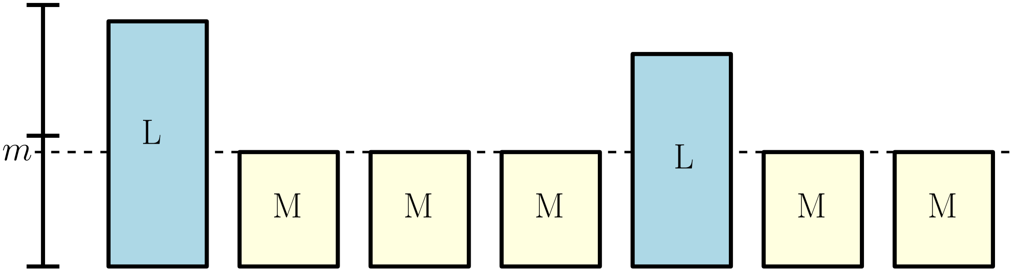


Beyond single medium item



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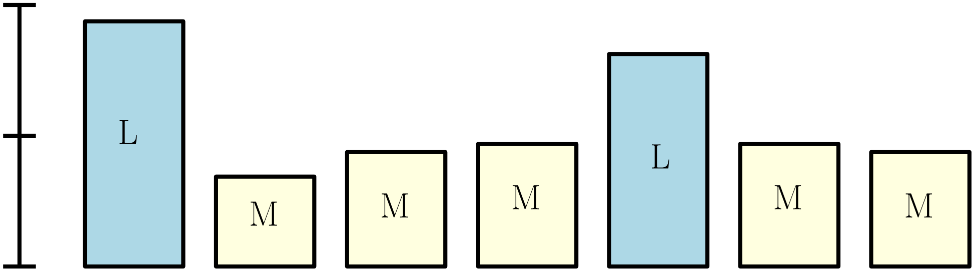
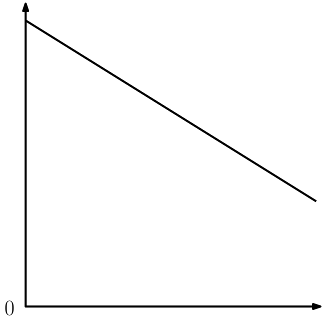


Beyond single medium item size



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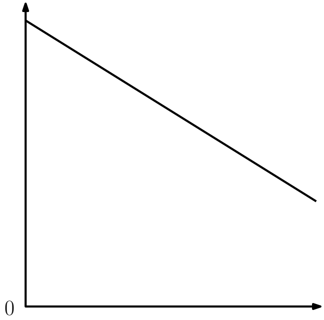
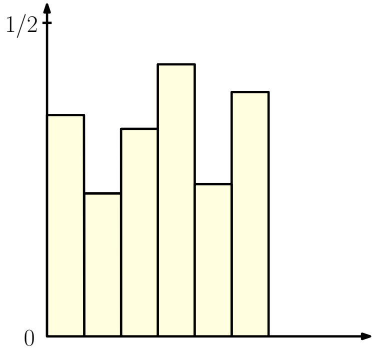


How many should wait? For different m, different answer!

Beyond single medium item size

- Sort waiting medium items
- Incoming medium item waits if fits below the curve



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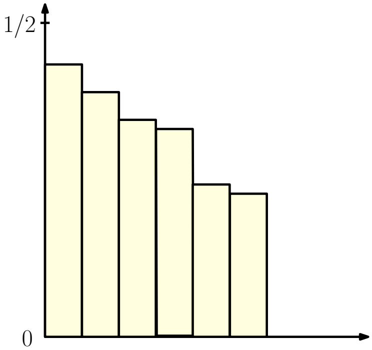

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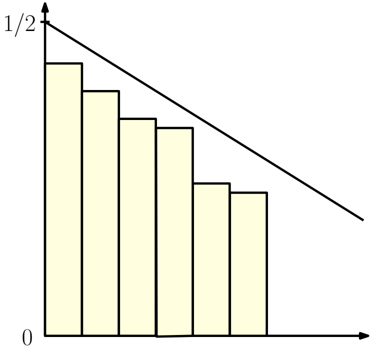

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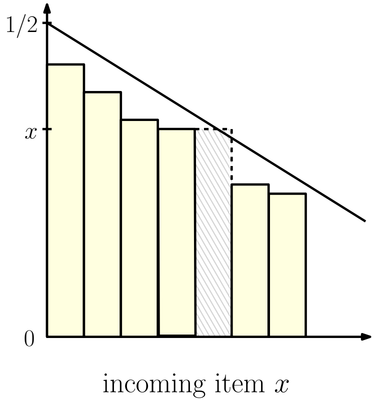
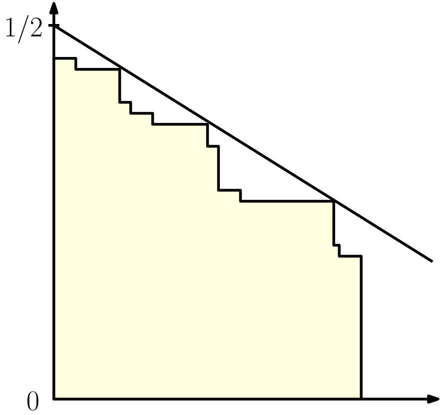


Analysis



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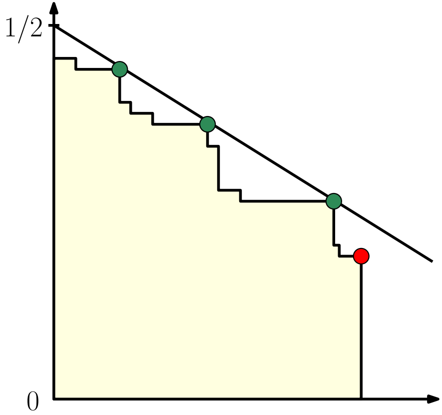
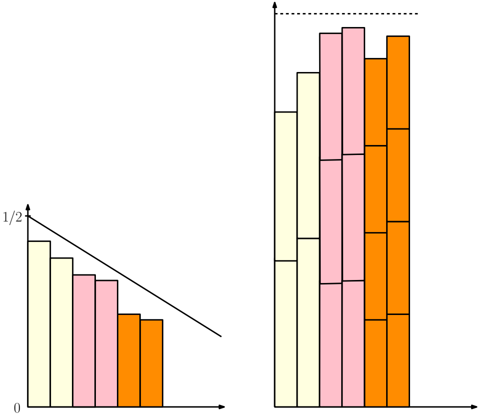


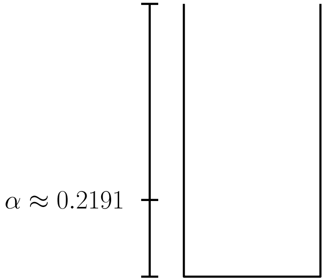
Possible to extend for $(\alpha, 1]$

$(\alpha \approx 0.2192)$



Next steps: algorithm for small items $(0, \alpha]$

just the idea – simply stacking small items is not enough



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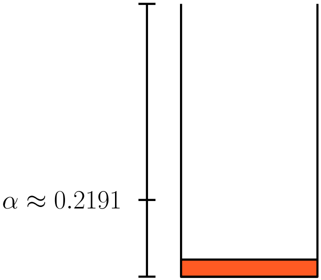

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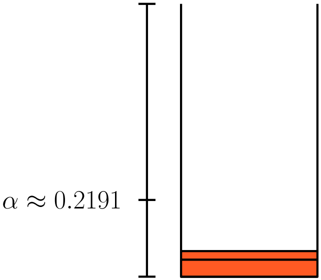

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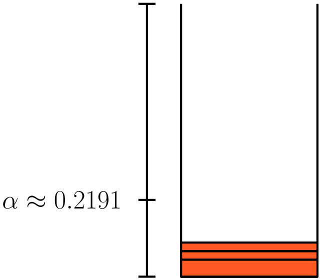

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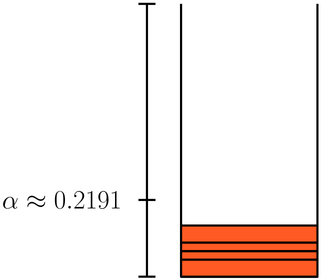

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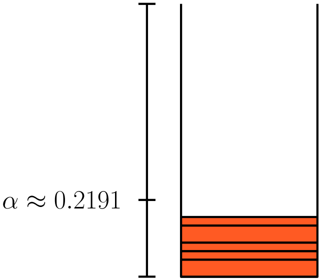

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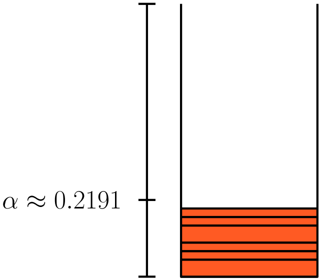

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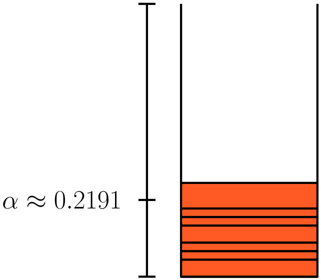

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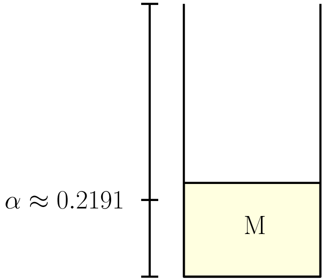
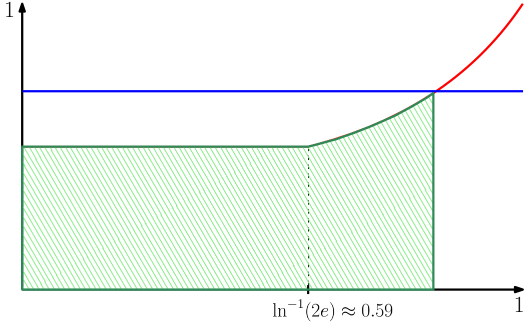


Rising Threshold Algorithm is optimal for Online Knapsack

and the function

$$f(x) = \max\{1/2, (2e)^{x-1}\}$$

is natural for this problem



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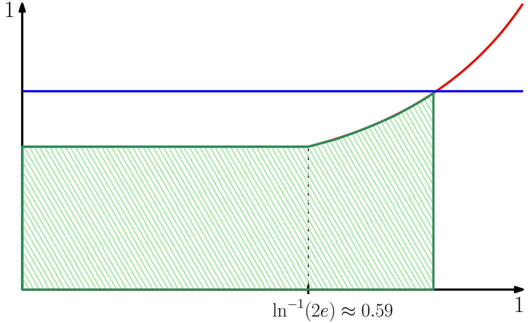

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